

# ANALYTICAL AND NUMERICAL MODELING OF ACOUSTIC MICROSYSTEMS

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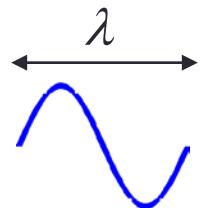


# Applications

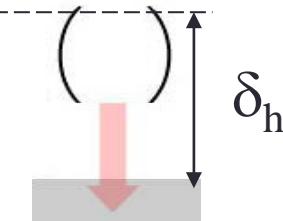
- Small acoustic elements (tubes, cavities,...)
- Electroacoustic transducers (MEMS or classical)
- Porous materials
- Acoustic metrology
- Thermoacoustic devices

# Thermal and viscous boundary layers

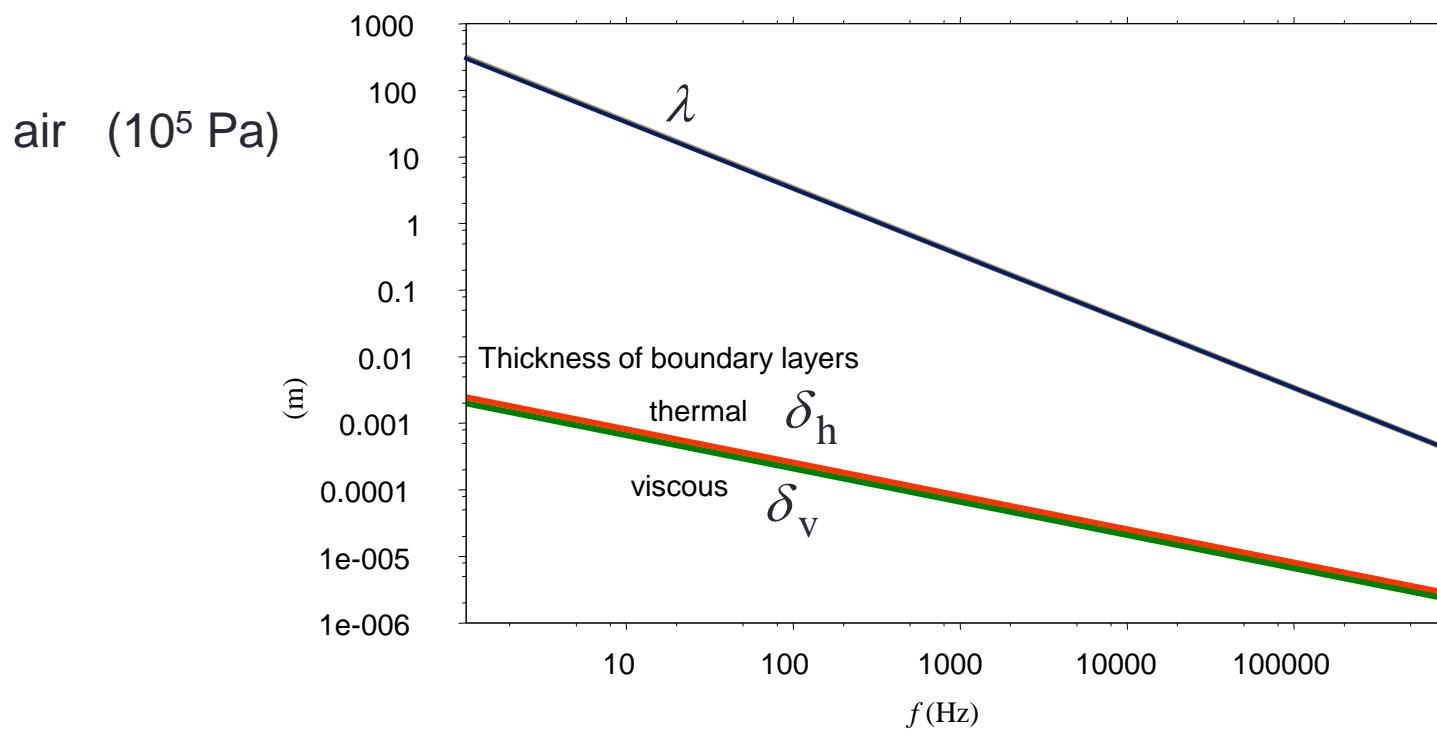
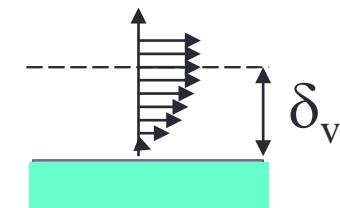
Acoustic wavelength



Thermal boundary layer



Viscous boundary layer



# Basic equations

- Navier-Stokes equation

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} + \text{grad } p - (\eta + 4\mu/3) \text{ grad div } \mathbf{v} + \mu \text{ rot rot } \mathbf{v} = 0$$

- Fourier equation for heat conduction

$$\left( \frac{1}{c_0} \frac{\partial}{\partial t} - \ell_h \Delta \right) \tau = \frac{1}{c_0} \frac{\gamma - 1}{\beta \gamma} \frac{\partial p}{\partial t}$$

- Conservation of mass

$$\rho_0 c_0 \text{div } \mathbf{v} + \frac{\gamma}{c_0} \frac{\partial}{\partial t} (p - \beta \tau) = 0$$

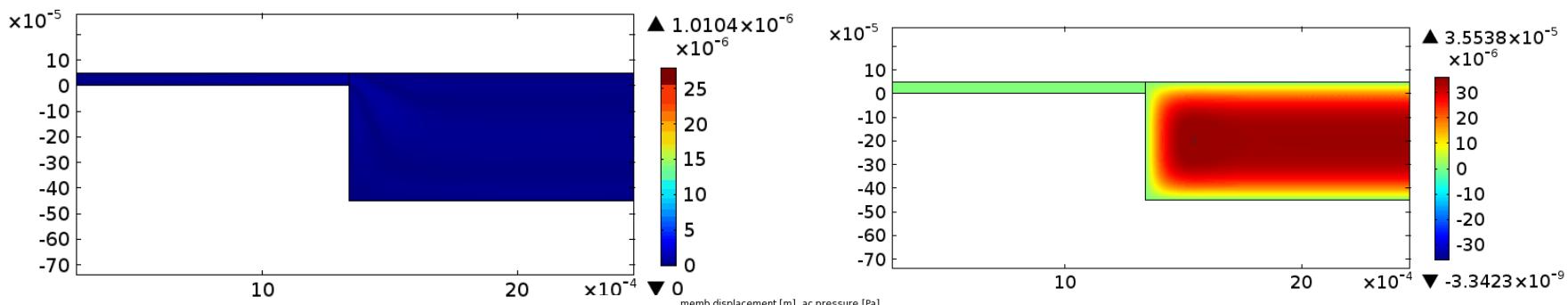
$\mathbf{v} = \mathbf{0}, \quad \tau = 0 \quad \text{on rigid isothermal boundaries}$

# Numerical modeling

- System of coupled equations for  $\boldsymbol{v}$  and  $\tau^*$

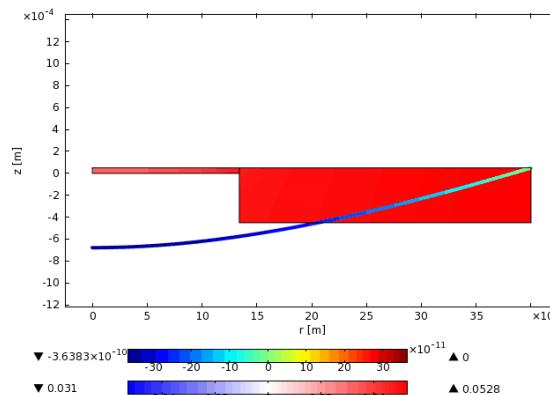
$$-\omega \boldsymbol{v} - \left( \frac{c_0^2}{\gamma} + j\omega c_0 \ell_v \right) \mathbf{grad} \operatorname{div} \boldsymbol{v} + j\omega c_0 \ell'_v \mathbf{rot} \mathbf{rot} \boldsymbol{v} + j\omega \frac{\hat{\beta}}{\rho_0} \mathbf{grad} \tau = \mathbf{0}$$

$$j\omega \tau - \gamma \ell_h c_0 \operatorname{div} \mathbf{grad} \tau + \frac{\gamma - 1}{\gamma \hat{\beta}} \rho_0 c_0^2 \operatorname{div} \boldsymbol{v} = 0$$



- Acoustic pressure

$$p = \hat{\beta} \tau - \frac{\rho_0 c_0^2}{j\omega \gamma} \operatorname{div} \boldsymbol{v}$$



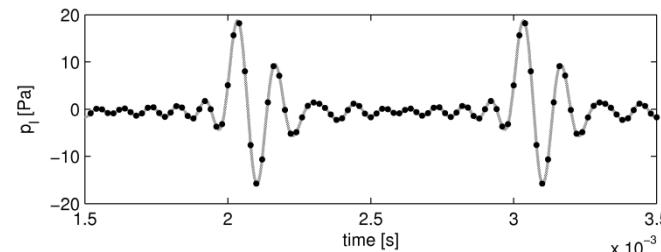
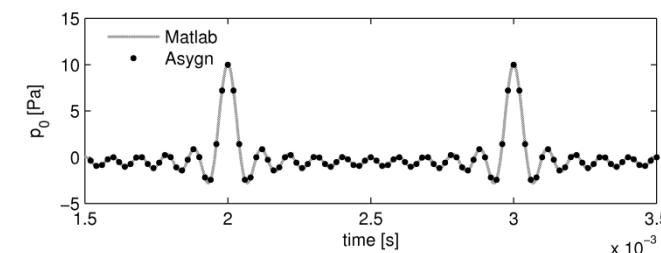
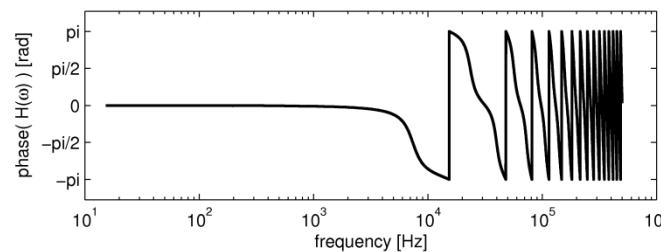
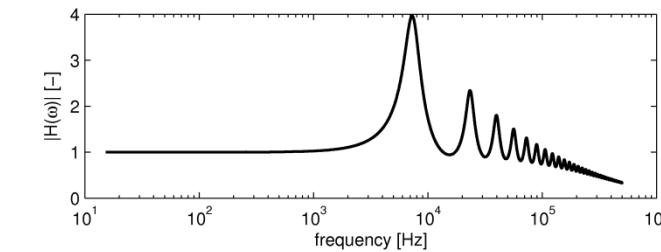
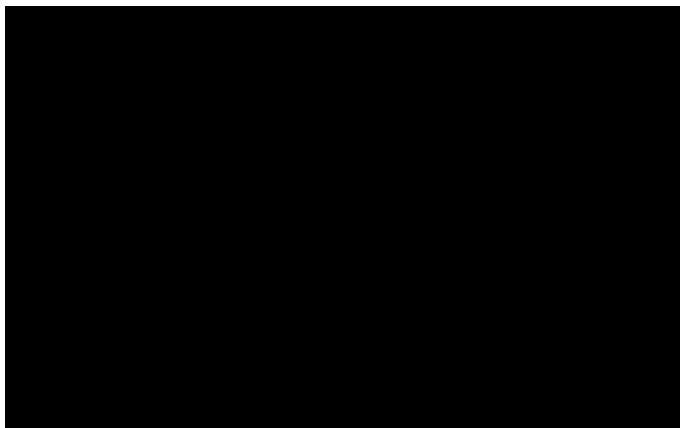
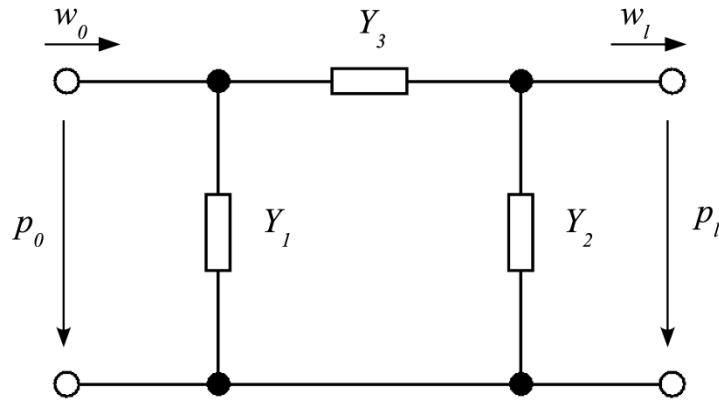
\* Joly et al.: Coupled equations for particle velocity and temperature variation as the fundamental formulation of linear acoustics in thermo-viscous fluid at rest, *Acta Acustica united with Acustica* 92, 202-209 (2006)

# Analytical modeling

- Solution of basic equations with several assumptions
  - small harmonic acoustic perturbations
  - quasi-plane wave approximation
  - one vector component negligible against the other  
(the same for derivatives)
  - mean value of the quantities across the element
- Lumped elements
  - Complex impedances
  - Limited series expansion of complex impedances  
=> acoustic compliances, masses, resistors

# Example 1: Small closed tube - lumped elements

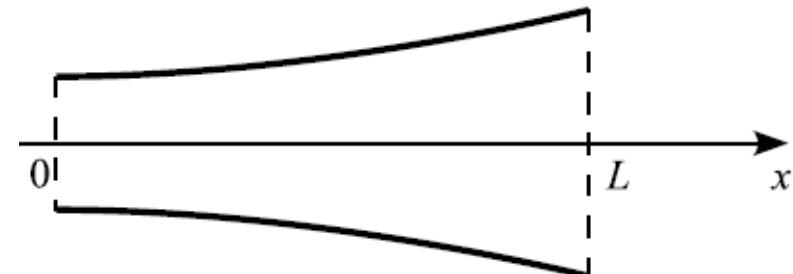
Radius:  $120\mu\text{m}$ , length: 10mm



# Example 2: Tapered small horns

Webster's horn equation

$$\left[ \partial_{xx} + \sigma(x) \partial_x + k_0^2 f(x) \right] p(x) = 0$$



Change of variable

$$x \rightarrow \xi = \int_0^x \frac{F_v(0)S(0)}{F_v(x')S(x')} dx' \Rightarrow \left[ \partial_{\xi\xi} + k_0^2 \phi(\xi) \right] p(\xi) = 0$$

Volterra integral equation

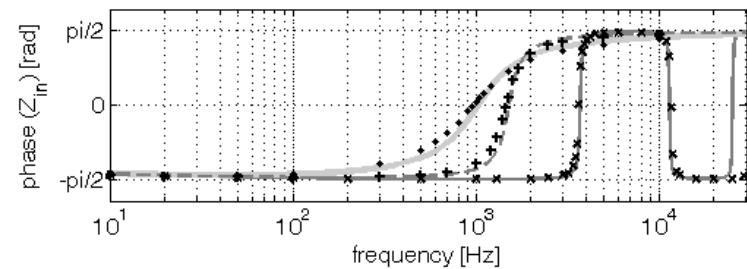
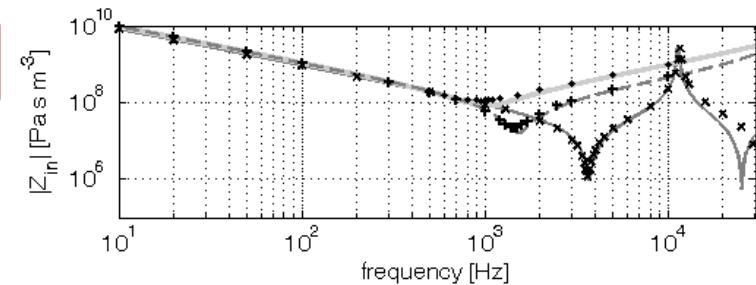
$$p(\xi) = p(0) + \partial_\xi p(0) \xi - k_0^2 \int_0^\xi (\xi - \zeta) \phi(\zeta) p(\zeta) d\zeta$$

Born approximation

$$p_0(\xi) = p(0) + \partial_\xi p(0) \xi$$

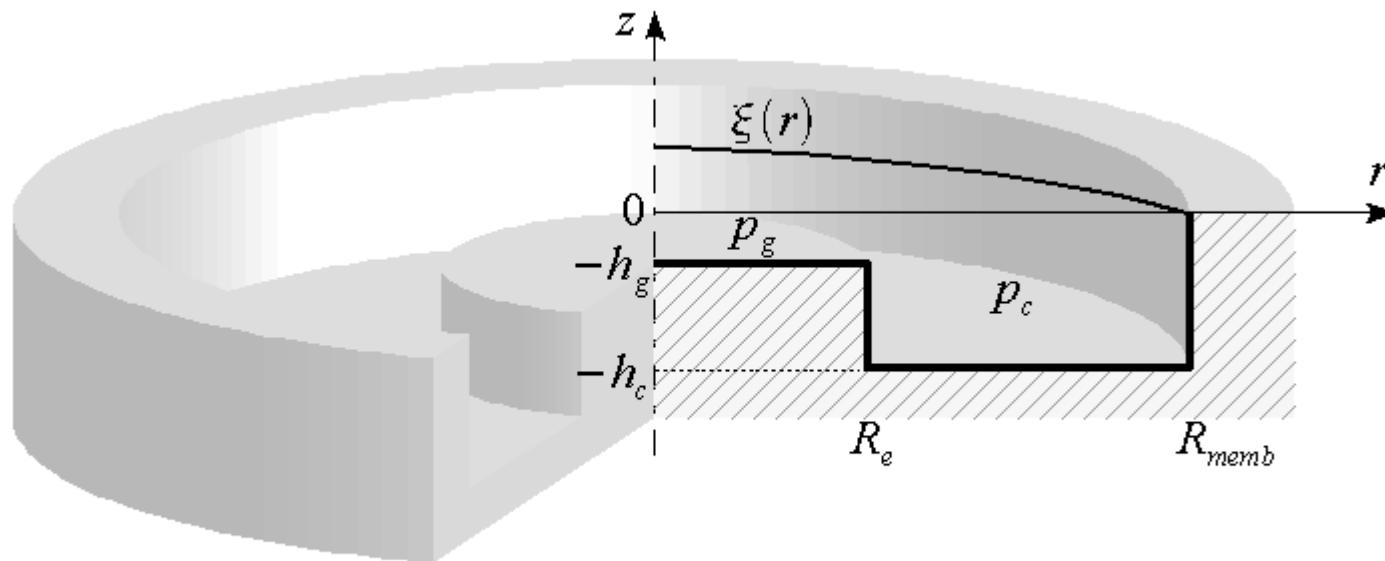
$$p_1(\xi) = p_0(\xi) - k_0^2 \int_0^\xi (\xi - \zeta) \phi(\zeta) p_0(\zeta) d\zeta$$

Example: small exponential horn



# Example 3: Electrostatic transducer

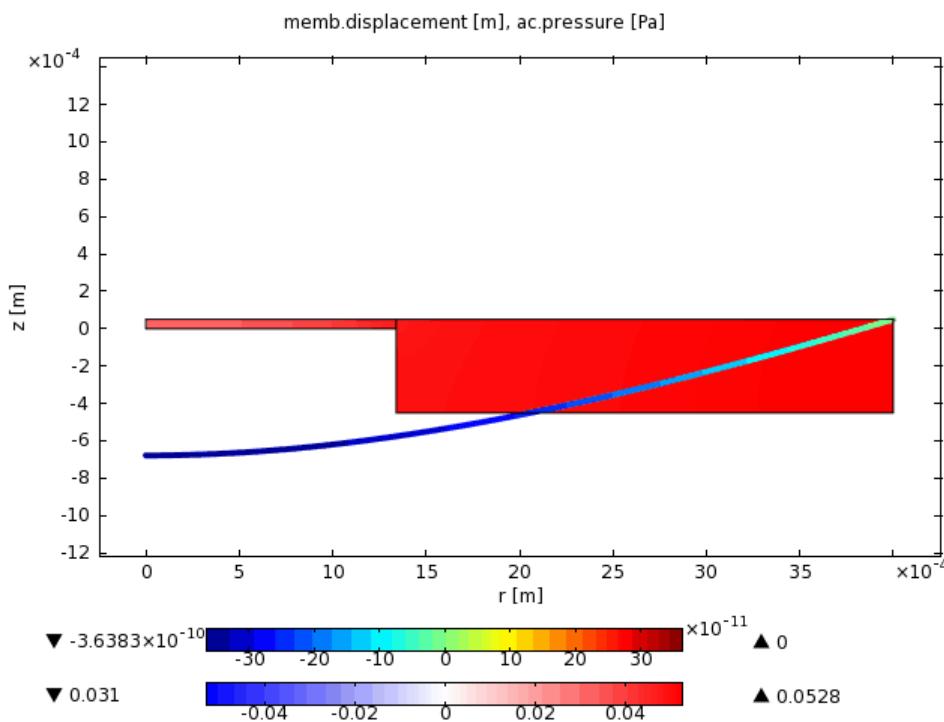
- Fluid – membrane coupling



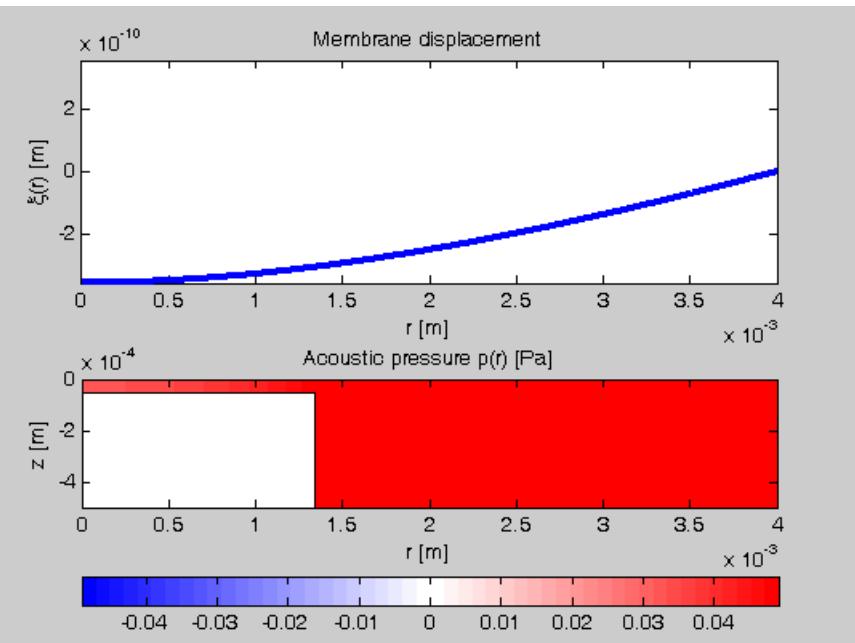
# Example 3: Electrostatic transducer

- Numerical and analytical results at 10 kHz

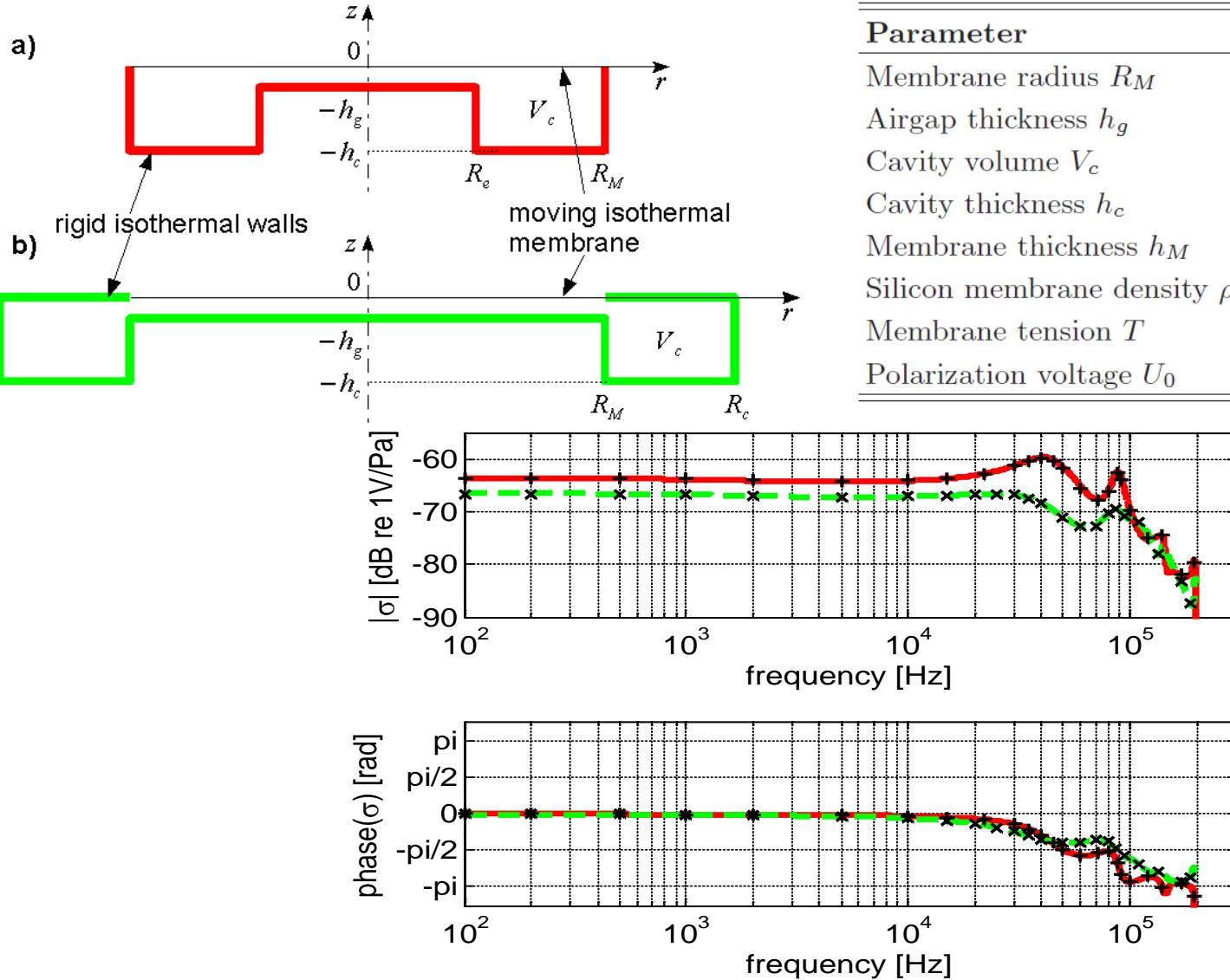
Numerical (Comsol)



Analytical (Matlab)



# Example 3: Electrostatic transducer



# Thank you for your attention

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