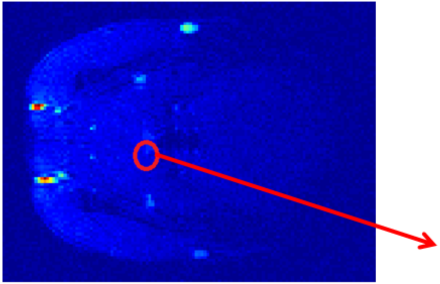
Utilization of compressed sensing in perfusion MRI

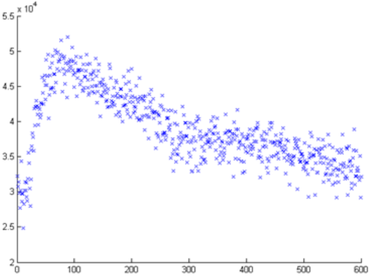
Marie Daňková

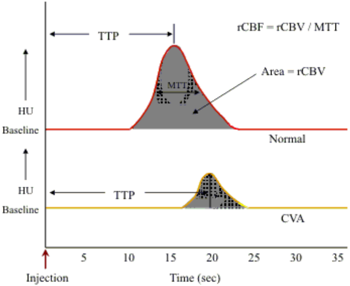
4^{th} SPLab Workshop

20^{th} November 2014

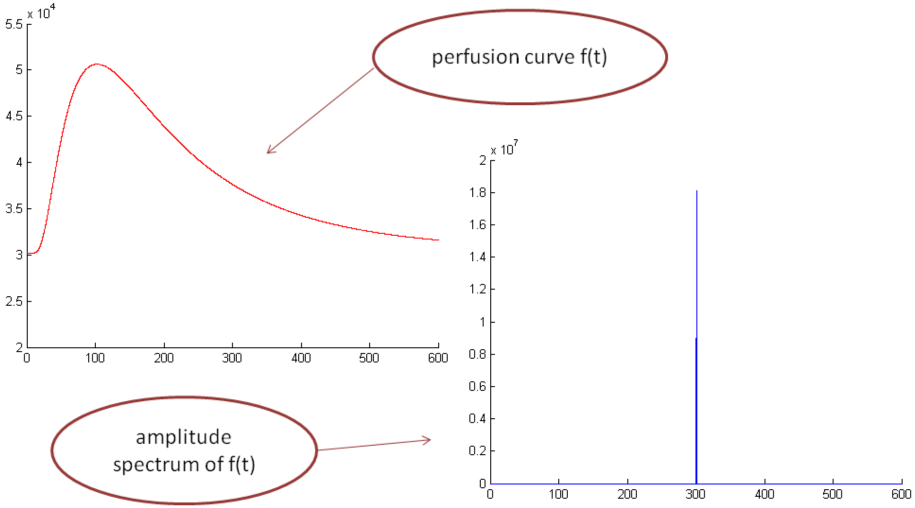
Perfusion imaging







Lognormal model

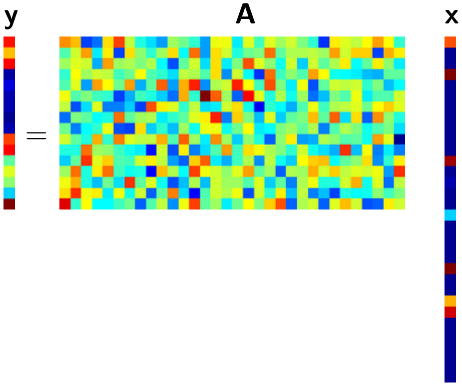


Sparse representation of signals

- model of signal $\mathbf{y}$:

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

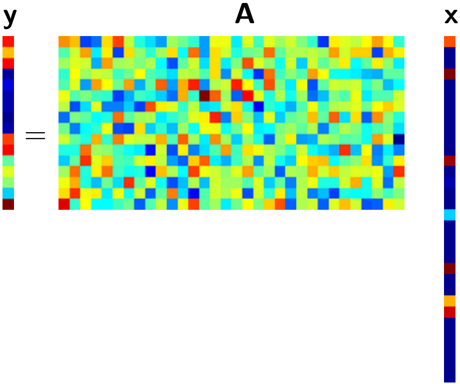
- infinitely many solutions of this equations



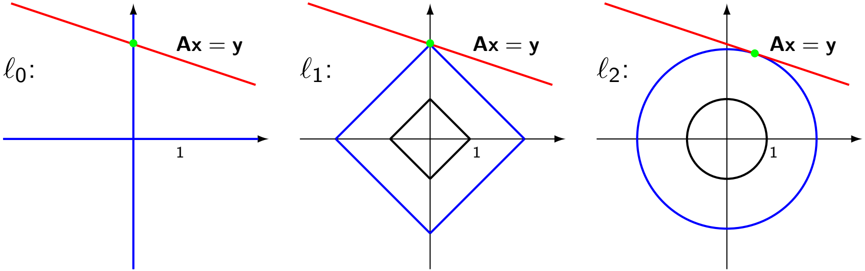
Sparse representation of signals

- We want the sparsest possible, i.e.

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y} \tag{P0}$$



ℓ_1-relaxation



- relaxed problem

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{Ax} = \mathbf{y}. \tag{P1}$$

Compressed sensing

- acceleration of the measurement
- longer computation time

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y}$$

a priori information
– sparsity of perfusion curve spectrum
– low rank matrix of video data

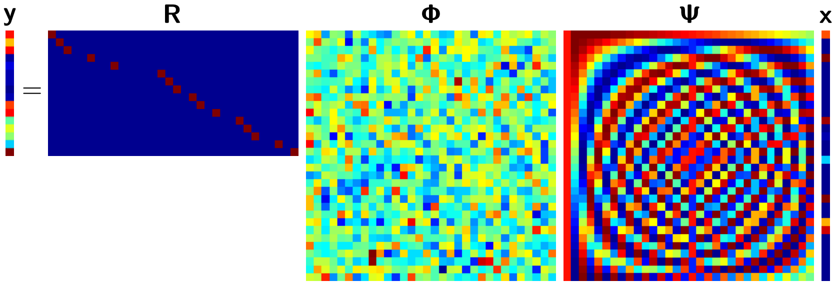
measurement

Compressed sensing

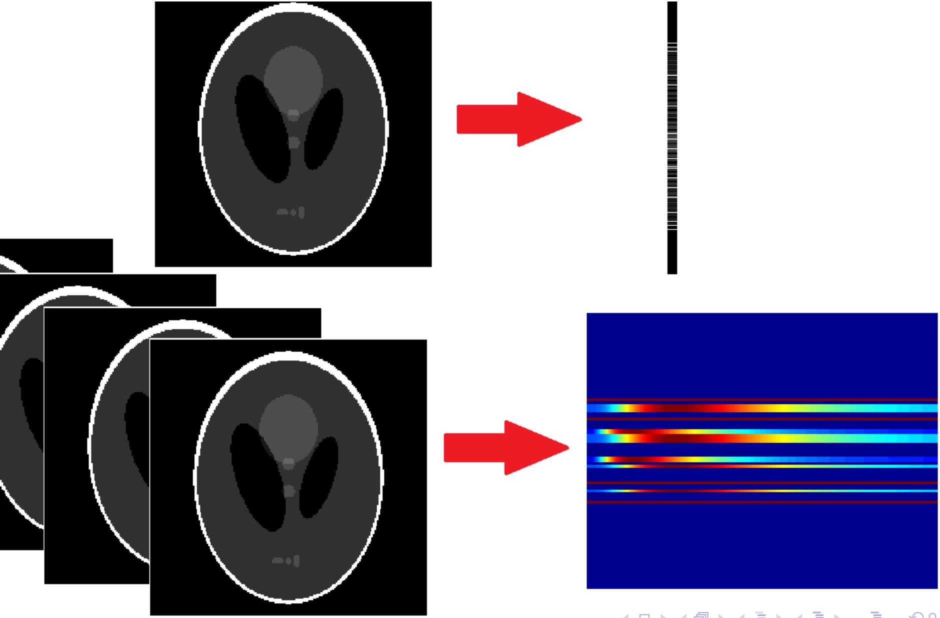
- CS problem:

$$\tilde{\mathbf{x}} := \arg\min \|\mathbf{x}\|_0 \quad \text{subject to} \quad \mathbf{y} = \rlap{$\overbrace{\phantom{\mathbf{R}\boldsymbol{\Phi}\,\boldsymbol{\Psi}}}^{\mathbf{A}}$}\underbrace{\mathbf{R}\boldsymbol{\Phi}}_{\mathbf{P}}\,\underbrace{\boldsymbol{\Psi}\mathbf{x}}_{\mathbf{z}} \qquad \text{(P1U)}$$

  - $\boldsymbol{\Psi}$ is orthonormal basis for signal $\mathbf{z}$, coordinates $\mathbf{x}$ are sparse
  - $\boldsymbol{\Phi}$ matrix $N \times N$ (in MRI Fourier transform)
  - $\mathbf{R}$ is random matrix performing selection of m rows (in MRI trajectory in k-space)



Reshaping video data



L+S model

Suitable constant (1% of maximal singular value of L)

Suitable constant (1% of maximal absolut value of TS)

Matrix sparse in spectrum of rows

$$\min_{L,S} \underbrace{\frac{1}{2} \| E \underbrace{(L+S)}_{M} - d \|_2^2}_{\text{Data fidelity term}} + \lambda_L \|L\|_* + \lambda_S \|TS\|_1$$

Desired reconstruction

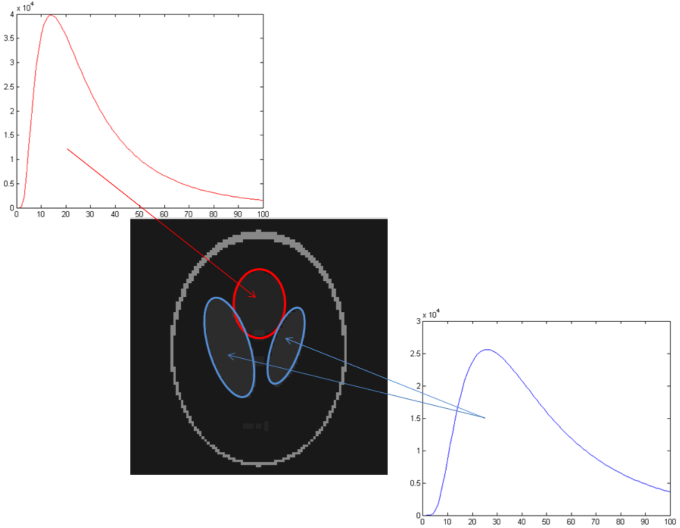
Low rank matrix

Measurement matrix + 2D Fourier transform

Measured data

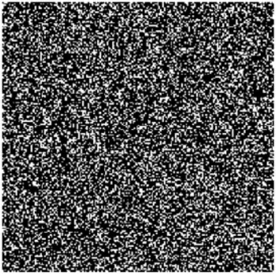
Fourier tr. for each row (perfusion curves have sparse spectrum)

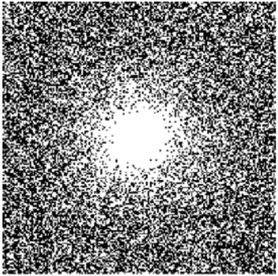
Simulation

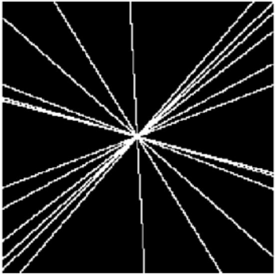


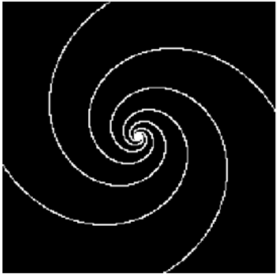
perfusion phantom – video

Measurement matrices

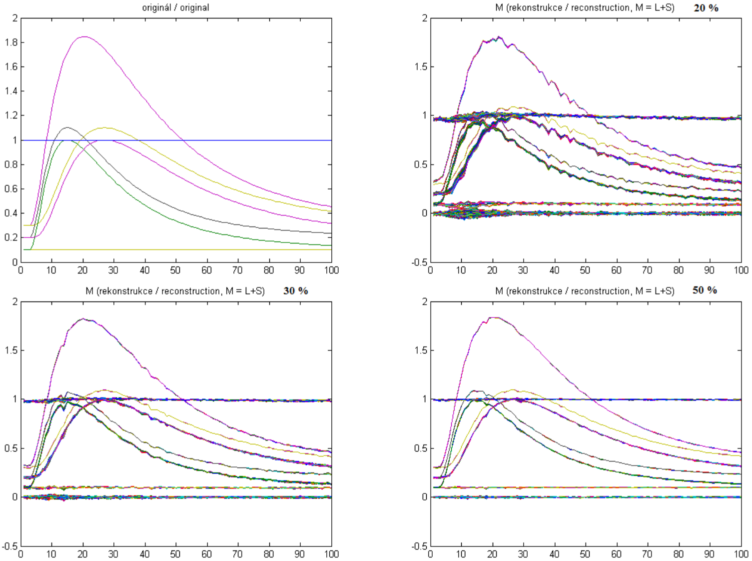






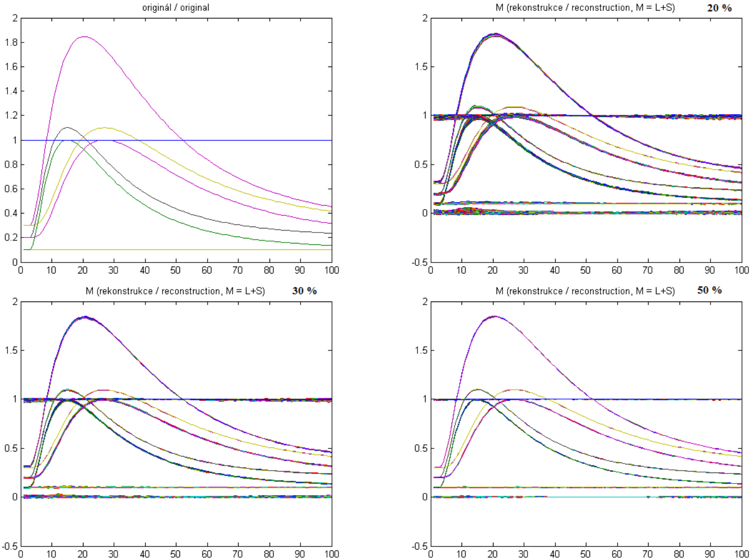


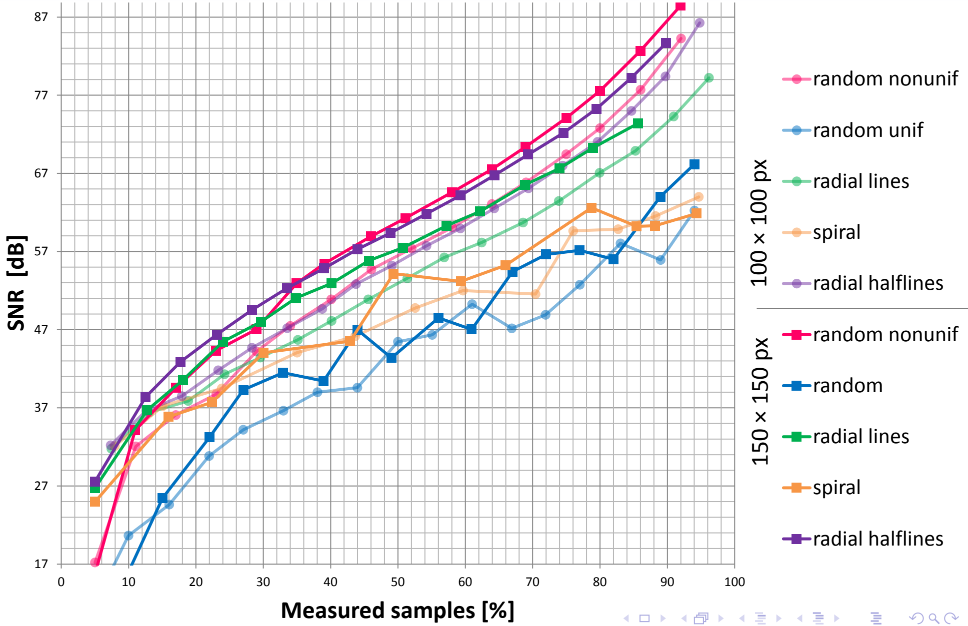
Random mask



originál / original
M (rekonstrukce / reconstruction, M = L+S) 20 %
M (rekonstrukce / reconstruction, M = L+S) 30 %
M (rekonstrukce / reconstruction, M = L+S) 50 %
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
-0.5 0 0.5 1 1.5 2
0 10 20 30 40 50 60 70 80 90 100

Radial halflines





SNR [dB]
Measured samples [%]
87
77
67
57
47
37
27
17
0
10
20
30
40
50
60
70
80
90
100
100 × 100 px
random nonunif
random unif
radial lines
spiral
radial halflines
150 × 150 px
random nonunif
random
radial lines
spiral
radial halflines

THANK YOU FOR YOUR ATTENTION

References

- R. Otazo, E. J. Candès and D. Sodickson. Low-rank and sparse matrix decomposition for accelerated dynamic MRI with separation of background and dynamic components. Submitted to *Magnetic Resonance in Medicine*, Sept. 2013.
- V. Harabis, R. Kolar, M. Mezl, R. Jirik. Comparison and Evaluation of Indicator Dilution Models for Bolus of Ultrasound Contrast Agents. Physiol Meas. 2013, 34(2)
- M. Mézl, R. Jiřík, V. Harabiš. Acquisition and data processing in the ultrasound perfusion analysis (in Czech) In New trends in biomedical engineering. Brno university of technology 2013, ISBN 978-80-214-4814-8.
- P. L. Combettes, J.-C. Pesquet. Proximal Splitting Methods in Signal Processing. *Fixed-Point Algorithms for Inverse Problems in Science and Engineering*. Springer, 2011.

References

- Lustig, Donoho, Santos, Pauly: Compressed Sensing MRI, IEEE Signal Processing Magazine, 2008
- Bruckstein, Donoho, Elad: From Sparse Solutions of Systems of Equations to Sparse Modeling of Signals and Images, SIAM Review, 2009
- Candés, Wakin: An Introduction to Compressive Sampling, IEEE, 2008
- http://fmri.mchmi.com
- http://spinwarp.ucsd.edu/neuroweb/Text/br-710epi.htm