ÖAW
Österreichische Akademie der Wissenschaften

Mathematics and Signal Processing in Acoustics
From Theory to Applications

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Overview:

Mathematics in Acoustics

Peter Balazs

The Acoustics Research Institute

Mathematics in Acoustics

Research Topics

Conclusions

Overview:

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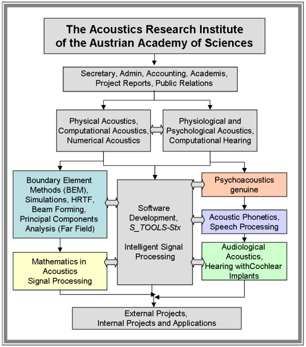
Overview:

The Acoustics Research Institute

Interdisciplinary approach to application-open fundamental research in acoustics.



22 employees. Part of the Austrian Academy of Sciences.

The Acoustics Research Institute

Work groups:

- Acoustic Phonetics and Phonology

  Speaker identification, Viennese Dialect, Speaker Models, ...
- Audiological Acoustics

  Binaural Hearing, Sound Localization, Cochlea Implants, Psychoacoustic Masking, ...
- Psychoacoustics

  Masking, Computational Auditory Scene Analysis, Pitch, Timbre, ...
- Computational Acoustics

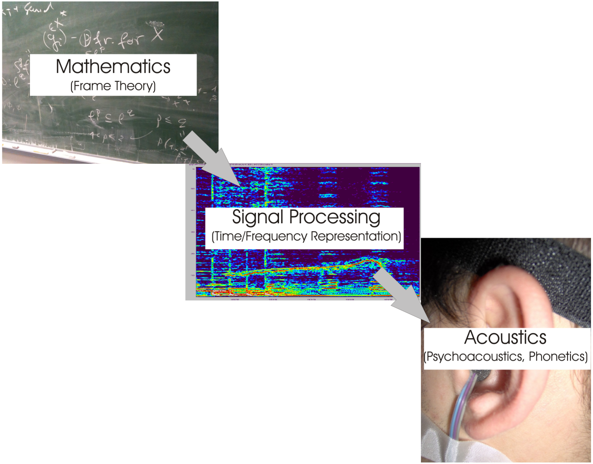
  Finite / Boundary Element Methods, Fast Multipole Method, Modal Analysis, Beam Forming, Acoustic Holography, Noise Barriers, ...
- Software Development

  ST^X, digital signal processing, sound database, ...
- Mathematics and Signal Processing in Acoustics

Mathematics and Signal Processing in Acoustics

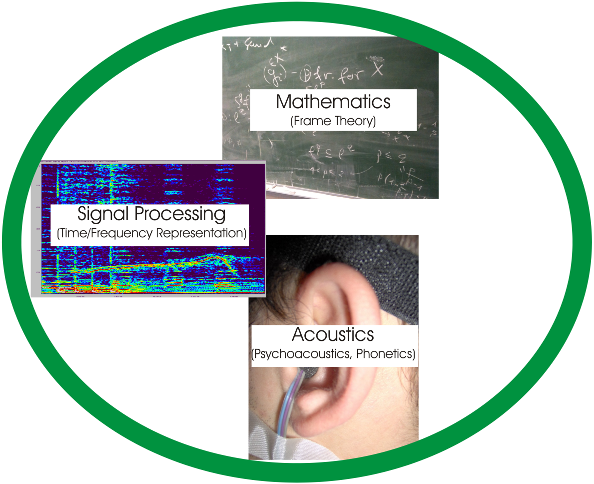
- **Founded 2008:** with a Viennese Science Foundation WWTF Project for 'High Potentials' (*'Frame Multipliers: Theory and Applications in Acoustics - MulAc-'*, March 2008- September 2011)
- **Growing:**
  - Currently 3 Researchers (funded by the Academy of Sciences),
  - 4 this November (FWF Lise Meitner project)
  - and 6 next May (START project *'Frames and Linear Operators for Acoustical Modeling and Parameter Estimation - FLAME-'*, 2012-2018)
- **Goal:**
  Application-oriented mathematics for acoustics with the focus on pluri-disciplinary research and internal, national and international cooperation.

State of the Art:



- Heuristic models
- Lacking precision
- Low efficiency
- Poor control of properties

Our Goal



- Application-oriented mathematics
- True inter-disciplinarity
- Synergy
- Novel methods for acoustics

(some) Research Topics

Frame Multipliers:

Analysis

$\Downarrow$

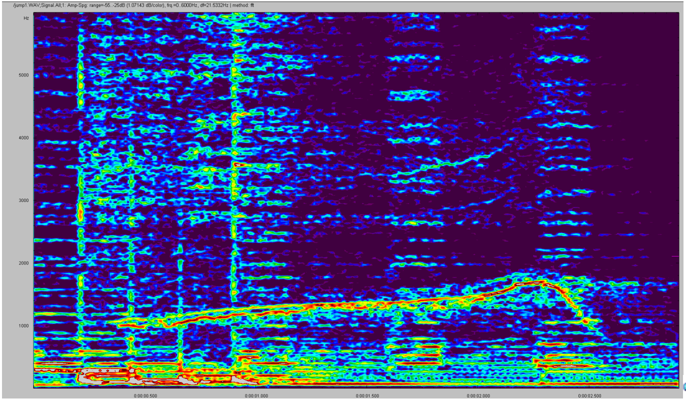
Multiplication

$\Downarrow$

Synthesis

Example for a Multiplier

Original audio file:

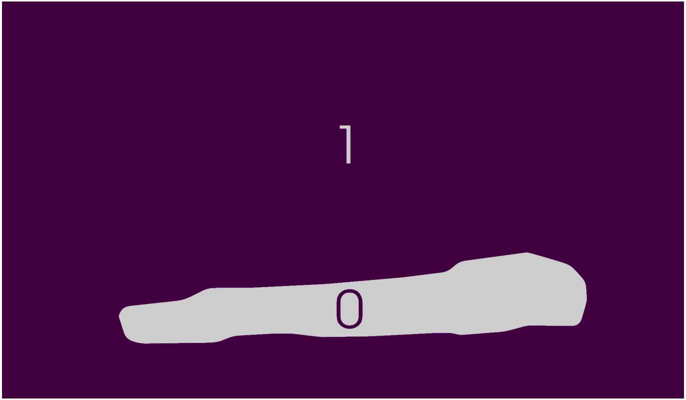


Example for a Multiplier

Mathematics in Acoustics

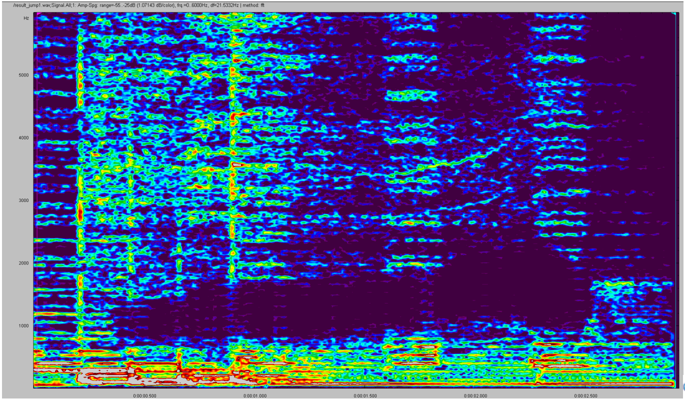
Peter Balazs

Symbol:



Example for a Multiplier

Result of Gabor Multiplier.



Frames

Definition

The sequence $(g_k | k \in K)$ is called a frame for the Hilbert space $\mathcal{H}$, if constants $A, B > 0$ exist, such that

$$A \cdot \|f\|_{\mathcal{H}}^2 \leq \sum_k |\langle f, g_k \rangle|^2 \leq B \cdot \|f\|_{\mathcal{H}}^2 \ \forall f \in \mathcal{H}$$

- Allows non-orthogonal expansion, redundant representation.
- Perfect reconstruction is guaranteed with the 'canonical dual frame' $\tilde{g}_k = S^{-1} g_k$ with S the frame operator (i.e. combined analysis/resynthesis operator).

$$f = \sum \langle f, \tilde{g}_k \rangle g_k = \sum \langle f, g_k \rangle \tilde{g}_k.$$

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$$f = \sum \langle f, \tilde{g}_k \rangle \, g_k = \sum \langle f, g_k \rangle \, \tilde{g}_k.$$

Frame Multipliers: Definition

Definition

Let $(\psi_k)_{k \in K}$, $(\phi_k)_{k \in K}$ be frames in the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$. Define the operator $\mathbf{M}_{m,(\phi_k),(\psi_k)} : \mathcal{H}_1 \to \mathcal{H}_2$, the frame multiplier, as the operator

$$\mathbf{M}_{m,(\phi_k),(\psi_k)} f = \sum_k m_k \langle f, \psi_k \rangle \phi_k$$

where $m \in l^\infty(K)$ is called the *symbol*.

ONS [Schatten, 1960]; regular Gabor frames [Feichtinger and Nowak, 2003]; general frames resp. Bessel sequences [Balazs, 2007]

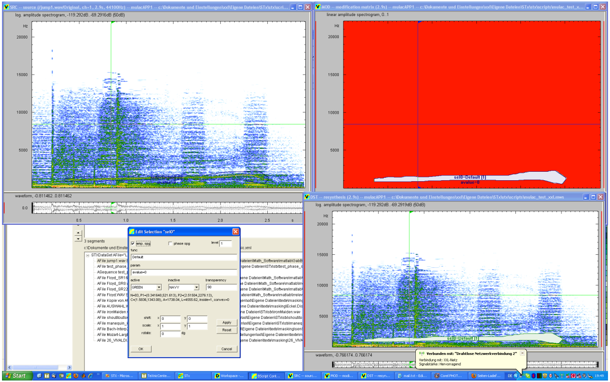
Theorem ([Balazs, 2007])

Let $\mathbf{M} = \mathbf{M}_{m,(\phi_k),(\psi_k)}$ be a frame multiplier for (ψ_k) and (ϕ_k) with the upper frame bounds B and B' respectively. Then

1. *If $m \in l^\infty$, then $\mathbf{M}$ is a well defined bounded operator.* $\|\mathbf{M}\|_{Op} \leq \sqrt{B'}\sqrt{B} \cdot \|m\|_\infty$.
2. $\mathbf{M}^*_{m,(\phi_k),(\psi_k)} = \mathbf{M}_{\overline{m},(\psi_k),(\phi_k)}$. *Therefore if m is real-valued and $\phi_k = \psi_k$ for all k, $\mathbf{M}$ is self-adjoint.*
3. *If $m \in c_0$, $\mathbf{M}$ is compact.*
4. *If $m \in l^1$, $\mathbf{M}$ is a trace class operator with* $\|\mathbf{M}\|_{trace} \leq \sqrt{B'}\sqrt{B} \cdot \|m\|_1$, *and* $tr(\mathbf{M}) = \sum_k m_k \langle \phi_k, \psi_k \rangle$.
5. *If $m \in l^2$, $\mathbf{M}$ is a Hilbert Schmidt operator with* $\|\mathbf{M}\|_{\mathcal{HS}} \leq \sqrt{B'}\sqrt{B} \cdot \|m\|_2$.

Also mathematical results for p-frames in Banach spaces, continuous frames, condition for invertibility for arbitrary sequences, etc...

Numerical results and algorithms



Implementation in STx and MATLAB, in the Linear Time-Frequency Analysis Toolbox (LTFAT) [Soendergaard et al., 2011] (available at Sourceforge) .

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Applications in Acoustics: Perceptual Sparsity by Irrelevance

MP3-Player

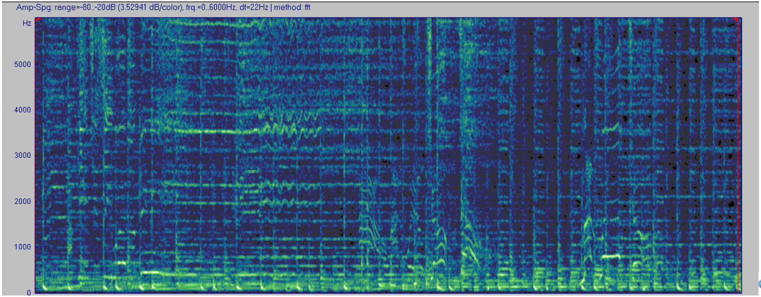
MP3:

- encoding / decoding scheme
- MPEG1/MPEG2 (Layer 3)
- signal processing
- psychoacoustical masking model

Perceptual Sparsity by Irrelevance

Existing algorithm in ST^X: simple model based on simultaneous masking, but effective algorithm!

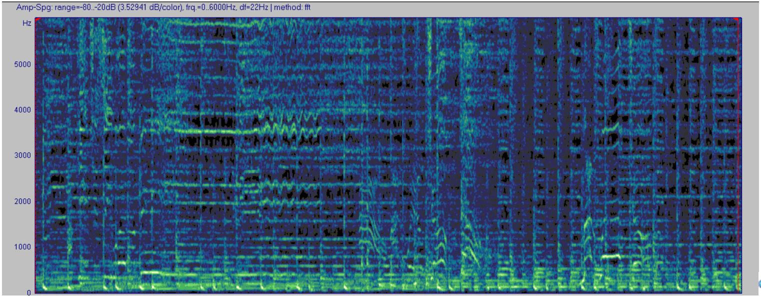
Original audio file



Psychoacoustic Masking

Existing algorithm in STx: simple model based on simultaneous masking, but effective algorithm!

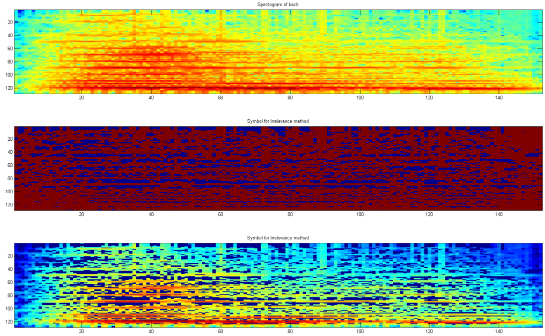
Irrelevance Filter



"Lossy Coding"

Perceptual Sparsity by Irrelevance

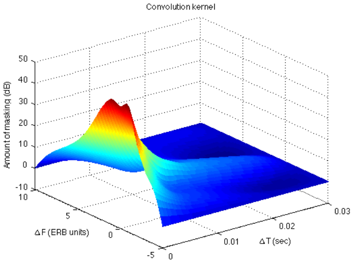
Interpreted as adaptive Gabor multiplier:



Irrelevance by Time Frequency Masking

Extend to True Time-Frequency Model using Multipliers:

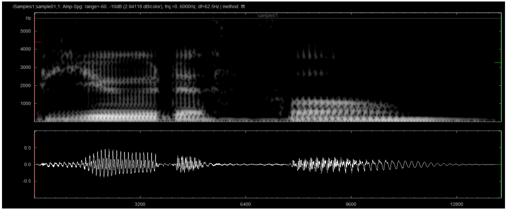
- Base it on non-stationary Gabor transform (NSGT) ([Balazs et al., 2011] $\rightarrow$ see talk by Monika), adapted to perception.
- Use new psychoacoustical data on time-frequency masking [Laback et al., 2011].



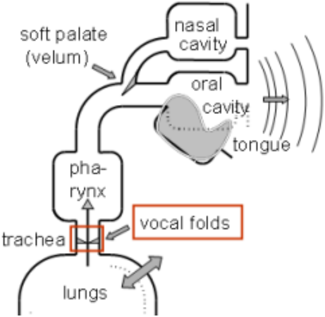
Long-term goal: Develop a new audio coder!

Speech and Speaker Analysis: using a Pole-Zero Model

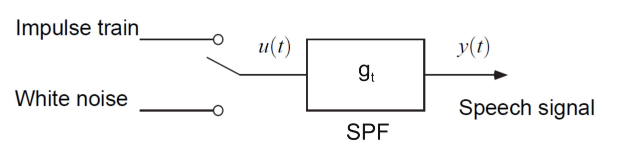
Digital Modeling of Speech



The goal: estimate the vocal tract from speech signal.



Digital Modeling of Speech



Goal: Estimate the speech production filter (SPF) $g(q)$ based on the observation of the speech signal $y(t)$ and the assumption on the excitation signal $u(t)$. See [Rabiner and Schafer, 1978, Markel and Gray, 1976, Quatieri, 2001].

Proposed Method: Estimation of Numerator and Denominator (ND, ND-FFT)

Assumption: Quasi-stationarity

Standard approach: All-Pole Model estimation on linear frequency scale.

Our approach [Marelli and Balazs, 2010]:

- Include Poles and Zeros!
- Take Psychoacoustics into account:
  - Minimize a logarithmic criterion
  - Use perceptual frequency scale
- Use an efficient, iterative algorithm (Quasi Newton-search using BFGS formula [Fletcher, 1987] for inverse of Hessian, analytic formulas for the Jacobian.)

Formant and Anti-Formant Tracking

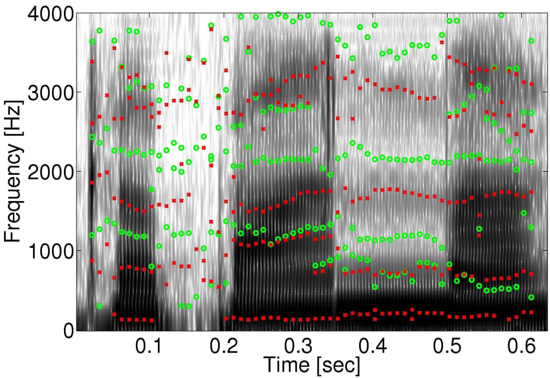


Figure: Poles (x) and zeros (o) of the SPF models estimated from the word "capanna".

Current and Future Work

- Importance for speaker verification (nasal and nasalized sounds).
- Link to physical models.
- Tracking algorithm applied on perception-motivated NSGT.
- Use for speech coding.
- Do a time-variant approach.

See `http://www.kfs.oeaw.ac.at/polezero`.

Improvement of Head-Related-Transfer-Function-Measurements

(see next talk)

Conclusions

Connecting mathematics, signal processing and acoustics leads to

- interesting mathematical results, as well as
- new methods and models for acoustics, as well as their implementation.

From Theory to Applications!

Thank you for your attention!

Questions? Comments?

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